Self-trapping as the possible beaming mechanism for FRB

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ABSTRACT

Mysterious Fast Radio Bursts (FRB), still eluding a rational explanation, are astronomical radio flashes with durations of milliseconds. They are thought to be of an extragalactic origin, with luminosities orders of magnitude larger than any known short timescale radio transients. Numerous models have been proposed in order to explain these powerful and brief outbursts but none of them is commonly accepted, it is not clear which of these scenarios might account for real FRB. The crucial question that remains unanswered is: what makes FRB so exceptionally powerful and so exceptionally rare?! If the bursts are related with something happening with a star-scale object and its immediate neighborhood, why all detected FRB events take place in very distant galaxies and not in our own galaxy!? In this paper we argue that the non-linear phenomenon - self-trapping - which may provide efficient but rarely occurring beaming of radio emission towards an observer, coupled with another, also rare but powerful phenomenon providing the initial radio emission, may account for the ultra-rare appearance of FRB.

Key words: Fast Radio Bursts - FRB

1 INTRODUCTION

Fast Radio Bursts (Lorimer et al. 2007; Thornton et al. 2013; Chatterjee et al. 2017) (hereafter referred as FRB) are spatially sporadic and temporarily intermittent radio emission outbursts of mysterious nature, happening throughout the universe, with duration of milliseconds. On the basis of a few credible observational arguments (e.g., observed dispersion measures greater than the maximum expected from the Galaxy, their spatial distribution mostly off the Galactic plane) it is strongly ascertained that FRB are most likely of extragalactic origin. This circumstance would necessarily imply that the radio luminosities of related astronomical sources are by several orders of magnitude larger than any previously detected millisecond-scale radio transient sources (Cordes & Wasserman 2016).

Originally FRB were detected with large radio telescopes, with localization accuracy of the order of a few arcminutes. Evidently, localization efforts have been made and were related with the survey of simultaneous variability of the immediate neighborhood, adjacent area galaxies (Keane et al. 2016) or possible presence of peculiar field stars (Loeb et al. 2014). Until recently these systematic and repeated efforts failed to pinpoint their location, to lead to the detection of precise sources of FRB or, at least, their host galaxies with a satisfactory level of accuracy.

However recently, by means of high-time-resolution radio interferometric observations, allowing direct imaging of the bursts *per se*, one of these source, FRB 121102, was localized with a sub-arcsecond accuracy (Chatterjee et al. 2017). It appeared to be related to a persistent and faint radio source with non-thermal continuum spectrum. It also appears to have a very faint, 25-th magnitude, optical counterpart. Evidently FRB 121102 remains quite exceptional: so far it turns out to be the *only* known *repeating FRB* (Spitler et al. 2014, 2016; ?; Scholz et al. 2016). Even if FRB 121102 is unique member of the 'family' of RRBs, still the repetitive nature of its bursts, makes less likely different kinds of 'catastrophic' scenarios, happening with an astronomical object once in a lifetime. Another, very important and noteworthy aspect of FRB, is that when it happens no enhancement of the radiation emission in any other spectral range has ever been detected.

Evidently, there are quite a number of different models of FRB. For instance, as early as in 2013 (Kashiyama et al. 2013), in order to explain four FRB reported in (Thornton et al. 2013), it was suggested that binary white dwarf mergers could lead to FRB. A birth of a quark star from a parent neutron star experiencing a quark nova was also suggested to be an explanation of FRBs (Shand et al. 2016). It was also suggested that FRB might be generated by 'cosmic bomb' - a regular pulsar, otherwise unnoticeable at a cosmological distance, producing a FRB when its magnetosphere is suddenly "combed" by a

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nearby, strong plasma stream toward the anti-stream direction (Zhang 2017). It was also argued that a black hole absorbing a neutron star companion on the the battery phase of the binary, when the black hole interacts with the neutron star magnetic field could become a source of at least a subclass of FRB (Mingarelli et al. 2015). However, this mechanism is expected to produce electromagnetic radiation mainly in the high-energy (X-rays and/or gamma-rays) range, while FRB are observed only in the radio range. In another interesting model (Falcke & Rezzolla 2014) FRBs are surmised to represent final signals of a supermassive rotating neutron star: it is supposed that initially they are above the critical mass for non-rotating models, supported by their rapid rotation. But magnetic braking constantly reduces their spins, and at some moment of time these neutron stars start suddenly collapsing to a black hole, producing a FRB. A somewhat similar model was suggested in (Fuller & Ott 2015): neutron star collapsing as a result of 'sedimentation' of dark matter (dark matter particles sinking to the center of a neutron star and becoming the same temperature as the star) within its core. Eventually, black hole is created at the center of the neutron star, with the collapse leading to the powerful radio outburst.

Obviously special attention is focused on the repeating FRB 121102 source. Its persistent radio counterpart is believed to have number density of particles of the order of $N \sim 10^{52}$, energy about $E_N \sim 10^{48}$ erg, and its length-scale of the order of $R \sim 10^{17}$ cm. The FRB source is argued to be a nebula heated and expanded by an intermittent outflow from a peculiar magnetar a neutron star powered not by its rotational energy but by its magnetic energy (Beloborodov 2017). The peculiarity of the object is related with its very young age; it is supposed to liberate its energy frequently, in giant magnetic flares driven by accelerated ambipolar diffusion in the neutron star core. The flares would eventually feed the nebula and produce bright millisecond bursts. In (Viyero et al. 2017) yet another model for repeating FRBs was proposed, implying the existence of a variable and relativistic electron-positron beam, being boosted by an impulsive MHD mechanism, interacting with a plasma cloud at the center of a dwarf galaxy. According to this model, the interaction leads to the development of plasma turbulence and creates areas of high electrostatic field - cavitons - in the cloud. It is argued that as a result short-lived, bright coherent radiation bursts, FRB, are generated.

Summarizing, we can cite a very recent review paper by J. I. Katz, where he says 'More than a decade after their discovery, astronomical Fast Radio Bursts remain enigmatic. They are known to occur at 'cosmological' distances, implying large energy and radiated power, extraordinarily high brightness and coherent emission. Yet their source objects, the means by which energy is released and their radiation processes remain unknown'. (Katz 2018).

We believe that before trying to involve exotic phenomena for the explanation of FRB it is reasonable to try to explain FRB on the basis of traditional, well-established physical phenomenon. In this letter, we argue that a well-known nonlinear optical phenomenon - self trapping (Chiao et al. 1964, 1965) - could serve as an alternative FRB model. The advantage of the proposed model is in its self-sustained and autonomous nature: it doesn't require additional sources of energy and it naturally provides sufficiently high emission in

a narrow spectral range without leading to a simultaneous radiation outburst in any other spectral ranges. It is also worthwhile to note that outbursts of similar nature are observed for certain objects in our galaxy, for instance, gamma-ray bursts for the Crab Nebula (Buhler & Blandford 2014). Recently for the explanation of these powerful bursts the elements of nonlinear optics (Machabeli et al. 2015) have been used.

Before coming directly to the core of the problem and the contents of our model let us note that nonlinear optics is based on the fundamental principle of self-focusing of a powerful electromagnetic wave passing through a nonlinear medium. The self-focusing effect is related with the dependence of the medium dielectric permittivity on the wave intensity. A good example of a nonlinear medium is a liquid/plasma/gas which under the influence of a powerful electromagnetic wave develops coherent orientation of its molecules along the field. It leads, in its turn, to the anisotropy of the medium, increase of the electric field and the growth of the refraction index. In these circumstances the medium behaves as a focusing lens for incoming electromagnetic waves with transverse intensity gradient. The details of the mechanism are considered in the next section of the paper, while discussion of the model and its implications in the context of FRB phenomenon are given in the final section.

2 MAIN CONSIDERATION

Let us consider a cloud of relativistic electron-proton plasma which sustains powerful electrostatic Langmuir waves (plasma oscillations) of electrons relative to heavy ions. Their wavelength can not be less than Debye radius, which for a relativistic plasma is:

$$R_D = c\gamma_p^{3/2}/\omega_p, \tag{1}$$

where $\omega_p = (4\pi e^2 n_p/m)^{1/2}$ is Langmuir (plasma) frequency of nonrelativistic plasma, while γ_p and n_p are Lorentz factor and number density of particles, respectively, and c is the speed of light.

The spectrum of Langmuir waves in relativistic plasma has the form

$$\omega = \sqrt{\omega_p^2/\gamma_p^3 + 3\mathbf{k}^2c^2} \tag{2}$$

where $\mathbf{k}$ is the wavenumber vector of the electrostatic wave. The factor 3 is related with the spatial isotropy of the medium. For a magnetized plasma the isotropy is violated and instead of the factor 3 in (2) we have 1.

For Langmuir waves the Debye radius R_D is the charge separation length-scale and is defined by the distance at which electron density fluctuation can be shifted on the plasma oscillation period time-scale due to the thermal motion of electrons. It leads to the polarization of the medium in the Debye volume caused by the grouping of charged particles. If one knows the value of the Debye radius and the average distance between the particles $\langle d \rangle = n_p^{1/3}$, then it is possible to estimate the number of dipoles $(N_d = (R_D/\langle d \rangle)^3)$ in the Debye volume. We assume that the plasma cloud contains a large number of Debye volumes but only in one of them the radiation is directed along the line of sight. Further,

we suppose that the current of charged particles is continuous. Relativistic particles rapidly leave the volume but they are substituted by other, identical particles.Therefore the onset of the system with Langmuir waves can be considered quasi stationary. It allows us to use the dipole approximation.

Let us now suppose that the Debye volume is filled with spatially aligned dipoles, oriented towards the line of sight. Obviously, it is an idealization because, in reality, in the Debye volume, only a part of the dipoles could be aligned with the line of sight. Furthermore, let us also assume that through the Debye volume an electromagnetic wave of radio frequency is passing. Let us assume that its energy is less than the energy of the electrostatic waves, but nevertheless it is powerful enough to cause the shifting of the multitude of dipoles.

The polarization vector has the following form:

$$\mathbf{P} = eN_D\mathbf{r} \tag{3}$$

Even when the electric field $\mathbf{E}(t)$ of the incident wave is small it still manages to shift charged particles by a small displacement value $\mathbf{r}(t)$. The shifting, in its turn, causes the appearance of the restoring force $\mathbf{f}(t) = -\eta\mathbf{r}(t)$, where η ia an analogue of the spring constant in Hooke's law. However, when the field $\mathbf{E}(t)$ is not too small the displacement $\mathbf{r}(t)$ can be more considerable and the expression for the restoring force will contains also a second, nonlinear term:

$$\mathbf{f}(t) = -\eta\mathbf{r}(t) - q\mathbf{r}^3(t) \tag{4}$$

where q is a constant coefficient. Its value does not have a decisive role in the framework of the present consideration.

The value of the electron displacement $\mathbf{r}(t)$ can be determined from the equation of motion (Machabeli et al. 2015). In a relativistic case it has the following form:

$$m\gamma^3\frac{d^2\mathbf{r}}{dt^2} = -m\gamma\Gamma\frac{d\mathbf{r}}{dt} - \eta\mathbf{r} - q\mathbf{r}^3 + e\mathbf{E} \tag{5}$$

where γ is corresponding Lorentz factor and where the first term on the right hand side of the equation describes dissipation, with Γ being the damping rate.

Taking into account the definition (3) of the polarization vector $\mathbf{P}$ from (4) we derive:

$$\frac{d^2\mathbf{P}}{dt^2} + \Gamma\frac{d\mathbf{P}}{dt} + \left(\frac{\omega_0}{\gamma}\right)^2\mathbf{P} + Q\mathbf{P}^3 = \left(\frac{e^2N_D}{m\gamma^3}\right)\mathbf{E} \tag{6}$$

where $Q \equiv q/me^2N_p^3\gamma^3$ and ω_0, in this case, is the Langmuir oscillation frequency $\omega_0 = \omega_p$.

We have noted the incident wave's electric field $\mathbf{E}$ is supposed to be large, but still much less than the intensity of the internal field within the cloud. In this case the nonlinear term in (6) can be considered to be small and we can solve the equation by means of the method of successive approximations. In particular, supposing $\mathbf{P} \equiv \mathbf{P_L} + \mathbf{P_{NL}}$, with $\mathbf{P_L} \gg \mathbf{P_{NL}}$, and neglecting the nonlinear term we obtain:

$$\frac{d^2\mathbf{P_L}}{dt^2} + \Gamma\frac{d\mathbf{P_L}}{dt} + \left(\frac{\omega_0}{\gamma}\right)^2\mathbf{P_L} = \left(\frac{e^2N_D}{m\gamma^3}\right)\mathbf{E} \tag{7}$$

If we further write down the electric field of the wave as

$$\mathbf{E}(t) = \mathbf{A}\cos(\omega t) \tag{8}$$

then the solution of (7) is found to be:

$$\mathbf{P_L}(t) = \left(\frac{e^2N_D}{m\gamma^3\sqrt{(\omega^2 - \omega_0^2)^2 + 4\Gamma^2\omega^2}}\right)\mathbf{A}\cos(\omega t + \Phi) \tag{9}$$

where $\tan(\Phi) = \Gamma\omega/(\omega^2 - \omega_0^2)$.

Note that the Langmuir frequency $\omega \gg \omega_0$ is the frequency of radio emission. Therefore, the range of frequencies we consider is far from the resonance: $|\omega_0^2 - \omega^2| \gg 4\Gamma^2$ and the impact of the dissipation term can be neglected. The vector of polarization $\mathbf{P}$ is related to the electric field through the polarization of the medium μ in the following way: $\mathbf{P} = \mu\mathbf{E}$. This expression can be written as:

$$\mathbf{P}(t) = \mu(\omega)\mathbf{E}(t) \tag{10}$$

After this linear solution is found the equation in the nonlinear approximation has the following form:

$$\frac{\partial^2\mathbf{P}_{NL}}{\partial t^2} + \omega_0^2\mathbf{P}_{NL} = -\left(\frac{q\mu^3(\omega)}{m\gamma^3e^2N_D^2}\right)\mathbf{E}^3(t) \tag{11}$$

Let us rewrite $\mathbf{E}^3(t)$ using trigonometric identity $cos^3(\omega t) = (1/4)[3cos(\omega t) + cos(3\omega t)]$. Then on the right hand side of (11) we have two terms, describing the contribution of the first and the third harmonics. Accordingly one can write:

$$\mathbf{P}(t) = \mu(\omega, A)\mathbf{E}(t) \tag{12}$$

with $\mu(\omega, A)$ defined by: $\mu(\omega, A) = \mu(\omega)\left[1 + \frac{3q\mu^2(\omega)A^2}{4mn^2e^2\omega^2}\right]$. while $\mu(\omega)$ is determined from the solution of (7):

$$\mu(\omega) = \frac{e^2N}{m\gamma^3\omega^2} \tag{13}$$

Note that in the serial expansion of $\mu(\omega, A)$ only first non-vanishing terms are maintained.

The dielectric permittivity of the medium is described by the tensor $\varepsilon_{ij}(\omega, A)$. The connection between $\varepsilon_{ij}(\omega, \mathrm{E})$ and $\mu_{ij}(\omega, \mathrm{E})$ tensors is given by:

$$\varepsilon_{ij}(\omega, \mathrm{E}) = \delta_{ij} + 4\pi\mu_{ij}(\omega, A) \tag{14}$$

The induction vector $\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P}$, where $D_i = \varepsilon_{ij}(\omega, E)E_j$. Subsequently, taking into account(12) and (14), we write down Maxwell equation:

$$\nabla \times \mathbf{B} - (1/c)\left[\varepsilon(\omega) + \frac{3\pi q\mu^3(\omega)A^2}{mN^2e^2\omega^2}\right]\frac{\partial\mathbf{E}}{\partial t} = 0 \tag{15}$$

From this equation it is evident that the influence of the nonlinear term is equivalent to the change of the dielectric permittivity or the refraction index of the medium. When an electromagnetic wave is propagating in this medium the refraction index is $H = c/v_{ph}$ and it depends on the wave frequency. Hence, the dispersion of the electromagnetic radiation depends on the refraction index. From $H^2 = \varepsilon$ it turns out that the refraction index is equal to $H = H_L + H_{NL}$, where $H_L^2 = \varepsilon(\omega)$, while $H_{NL} \approx H_2A^2$ and for H_2 we have:

$$H_2 = 6\pi q\mu^3(\omega)/mN^2e^2\omega^2 \tag{16}$$

Therefore, if $H_2 > 0$ the refraction index in the cavern $H = H_L + H_{NL}$ turns out to be larger than the refraction index of the ambient beyond the Debye sphere (Machabeli et al. 2015), which remains equal to $H = H_L$.

Finally in the whole Debye volume let us separate rays directed to the observer. Due to the linear diffraction these rays has to diverge, feature angular diffusion across the the line of observation and before leaving the Debye volume they have to be confined within the cone with the opening angle $2\theta_D$ where

$$\theta_D \approx \frac{\lambda}{r_D H_L} \qquad (17)$$

where λ is the wavelength of the electromagnetic wave. However, when the rays leave the nonlinear medium and enter the ambient with the refraction index H_L, the rays experience nonlinear refraction. If the ray falls on the boundary between nonlinear, optically more dense medium and linear, optically less dense one and if the angle of incidence $\theta_0 > \theta_D$ then all diffracted rays will undergo a total internal reflection. We are interested in the regime when $\theta_0 \approx \theta_D$ when rays assemble in a parallel beam and the observer sees enhanced intensity of radiation (Machabeli et al. 2015). The limiting critical incidence angle for the total internal reflection is determined by the following equation:

$$cos\theta_0 = H_L/(H_L + H_2 A^2) \qquad (18)$$

For the small value θ_0 we find:

$$\theta_0^2 \approx 2(H_2/H_L)A^2 \qquad (19)$$

Substituting (16) in (18) we find out that

$$H_2 \simeq 1/\omega^8 \qquad (20)$$

and if the condition is satisfied for certain frequencies in a given region and at a given moment of time for other values of frequencies it would not hold. The frequency dependence is very strongly nonlinear, which implies that self-trapping will work only for a very narrow frequency range.

3   DISCUSSION AND CONCLUSIONS

In this paper we argue that the actual reason of FRB could be the self-trapping phenomenon. This nonlinear mechanism implies that to the radiation beam directed towards the observer additional rays are added which, in the absence of the self-trapping, would pass beyond the actual radiation pattern. As a result, the observer, while the beam is being self-trapped sees an enhanced intensity of radiation in a very narrow frequency range! This scenario is self-sustaining and fully autonomous because unlike many other mechanisms it does not require additional, external sources of energy. Besides, self-trapping depends on a quite large number of parameters, in particular, on the proper value of ratio of the wave amplitude to the amplitude of an incident electrostatic wave, on the temperature of the medium, direction of these waves relative to the line of sight. Any kind of, even a slight, deviation of any of these parameters from the "favorable" values may lead to the violation of the nonlinearity condition. That is why this is a very finely tuned, random and very rare phenomena. Its occurrence and the arrival of the self-trapped, self-focused enhanced beam to the observer has to be a totally random and extremely rare phenomenon.

Initially we select a volume, which, at the moment when they pass through a randomly appearing nonlinear medium. contains waves directed towards the observer. It can be said that rays in this volume constitute a cylindrical beam with maximum energy concentrated at its center. The area of maximum intensity at the same time is optically thicker one (Akhmanov et al. 1968). However the given volume, apart from the rays directed to the observer, contains also rays which propagate with some nonzero angle to the line of sight. Most of these rays, providing they pass only through a linear medium would not reach the observer. However, if these rays move from optically thicker area to optically less thick area they would be refracted towards the maximum energy area. Nonlinear area, selected by us, is significantly smaller than the plasma cloud in which the waves of the given frequency are generated. Therefore, it is reasonable to suppose that a significant number of the given frequency waves pass through the nonlinear region with propagation directions constituting small angles to the line of sight, which satisfy the condition of the total internal reflection at the boundary between the linear and nonlinear media. Evidently the coincidence of the channel axis with the line of sight has to be totally random. That is why FRB are happening rarely and on a totally random basis.

Even a slight alteration of these parameters leads to the violation of the self-trapping condition and, therefore, disappearance of the wave intensity enhancement - disappearance of the burst. If our model is correct and relevant to actual FRB, the self-trapping condition may hold only for a few milliseconds. Hence, it is reasonable to expect that the probability of the coincidence between the line of sight and the direction of self-trapping has to be quite small. Therefore, what could be a serious drawback for a commonly occurring phenomena in this case 'works' just the opposite way - it strengthens our confidence in believing that self-trapping could be the very reason of the appearance of this extremely rare and energetic phenomenon - fast radio burst or FRB. Additionally, self-trapping mechanism does not exclude other, physically plausible, repetitive or non-repetitive, catastrophic or non-catastrophic mechanism proposed for the FRB. Moreover, we believe that sel-trapping may be the very 'beaming' mechanism which might be needed for interpreting FRB as narrowly beamed radio bursts (Katz 2017). Any of those mechanisms giving credible explanation of rarity of FRB, coupled with self-trapping mechanism would imply the simultaneous occurrence of two, quite rare processes. Probably this is the very reason why FRB are not just rare, or very rare, but ultra-rare phenomenon, until now observed only from very distant, extragalactic sources.

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